

E(COMP.) / Sem-III / R-20 / C Scheme / DS.

(3 Hours)



Total Marks: 80

- N.B: (1) Question No. 1 is compulsory
 (2) Attempt any three questions out of the remaining five questions
 (3) Figures to the right indicate full marks
 (4) Make suitable assumptions wherever necessary with proper justifications

- Q1 A Differentiate between arrays and linked lists. [05]
 B Define data structure and Explain types of data structures. [05]
 C Write a program in C to reverse a string using stack. [05]
 D Explain the concept of a queue ADT and describe a real-world application where queues are useful. [05]
- Q2 A Write a program to perform following operations on the Doubly linked list [10]
 a) Create the list
 b) Insert element at the beginning
 c) Insert element at the end
 d) Delete last element of the list
 B Explain the process of deleting a node from a Binary Search Tree(BST). Illustrate the cases of deletion with examples. [10]
- Q3 A Write a program in C to convert a given infix expression into a postfix expression. [10]
 B Explain the concept of Huffman coding and describe the steps for constructing a Huffman tree. Given the following characters with frequencies, construct the Huffman tree and generate the Huffman codes: A:5, B:9, C:12, D:13, E:16 [10]
- Q4 A Construct an AVL tree by inserting the following elements in sequence, showing the tree after each insertion and rotation: 30, 20, 10, 25, 40, 50, 5, 6, 11, 12. [10]
 B Write a program in C to implement stack ADT using an array. [10]
- Q5 A Explain the key differences between a singly linked list, a doubly linked list, and a circular linked list. Use diagrams to show the structure of each type and discuss the advantages and disadvantages of each. [10]
 B Write a program in C to implement circular queue using linked list. [10]
- Q6 A Explain Depth First search and Breadth First search graph traversal techniques with example. [10]
 B A hash table has 10 slots, and the following keys are inserted in this order: 21, 33, 40, 98, 51, 64, 10, 75, 4, 86. Using linear probing as the collision resolution technique, show the final arrangement of keys in the hash table. Use the hash function $h(x) = x \% 10$. [10]

Time:

Marks: 80

- N.B. : (1) Question Number 1 is compulsory (2) Solve any three questions from the remaining questions
 3) Make suitable assumptions if needed
 4) Assume appropriate data whenever required. State all assumptions clearly.

1. a. Explain the following terms partition set with suitable example. 5
 1. Partition set
 2. Disjoint sets
- b. Construct the Truth Table and check if the following statement is tautology. 5
 $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$
- c. Let $f: A \rightarrow B$ be a Function from A to B. Prove that f^{-1} exists if and only if f is a Bijective Function. 5
- d. Prove by mathematical induction that $x^n - y^n$ is divisible by $x - y$ 5
2. a. Define Equivalence Relation. A relation R is called circular if aRb and bRc imply cRa . Show that R is circular if and only if it is an Equivalence Relation 8
- b. Let $A = \{1,2,3,4\}$ and $R = \{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)\}$. Find Transitive Closure of R using Warshall's algorithm. 8
- c. Let $f: R \rightarrow R$ be a function defined by $f(x) = 2x - 3$. Prove that it is Bijective & find inverse. 4
3. a. Let f, g, h be functions on real numbers R defined as follows: 8
 $f(x) = 2x + 5$, $g(x) = 5x + 3$, $h(x) = 3x$
 Find: 1) $f \circ g$ 2) $g \circ f$ 3) $g \circ h$ 4) $f \circ g \circ h$ 5) $g \circ f \circ h$
- b. Give the generating function for the following sequences 8
 1) $\{0,1,2,3,4,\dots\}$
 2) $\{1,2,3,4,5,\dots\}$
 3) $\{2,2,2,2,2,\dots\}$
 4) $\{0,0,0,1,1,1,\dots\}$



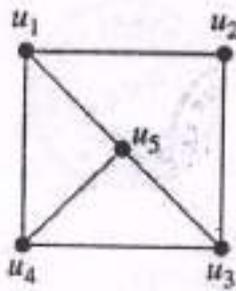
Qp code

Prog code
1T00733

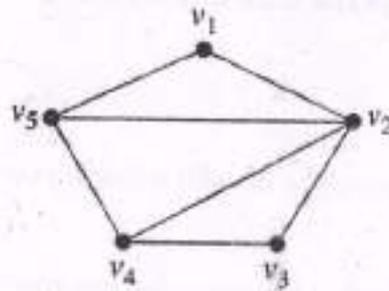
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- c Determine whether the following graphs are isomorphic. Justify your answer. 4



G1

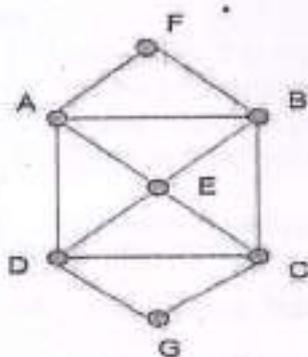


G2

- 4 a A Function $f: R - \{7/3\} \rightarrow R - \{4/3\}$ is defined as $f(x) = (4x - 5)/(3x - 7)$ g
 Prove that f is Bijective and find the rule for f^{-1}
- b Show that the $(2, 5)$ encoding function $e: B^2 \rightarrow B^5$ defined by 8
 $e(00) = 00000$ $e(01) = 01110$
 $e(10) = 10101$ $e(11) = 11011$ is a group code.
- c How many numbers must be selected from the set $\{1, 2, 3, 4\}$ to guarantee that at 4
 least one pair of these will add up to 7.



- 5 a Define Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit. Determine if the following diagram has Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit and state the path/circuit. 8



- b Prove that the set of Real numbers under $*$ defined by $a*b = a+b-2$ is a group 8
- c Find the complement of each element in D_4 4
6. a Draw the Hasse Diagram of D_{60} and check whether it is a Lattice. 8
- b Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$ 8
with initial conditions $a_0=1, a_1 = -1$
- c Define the following with suitable example. 4
a) Equivalence Class b) Sub group c) Distributive Lattice d) Injective Function



TIME: 03 HOURS

MAX. MARKS : 80

Note:

1. Question No. 1 is compulsory.
2. Attempt any three questions out of remaining five questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

- Q.1 Answer the following (Any four) Marks
- a. Find the Laplace transform of $t \sin^3 t$. 05
- b. Calculate the Spearman's rank correlation coefficient R. 05

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

- c. Find the constants a, b, c, d, e if $f(z) = (ax^3 + bx^2y + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + cxy + y)$ is analytic. 05
- d. Find inverse Laplace transform of $\tan^{-1} \left(\frac{s+d}{b} \right)$. 05

- Q.2 a. Evaluate by using Laplace transform of $\int_0^\infty \left(\frac{\sin 3t + \sin 2t}{te^t} \right) dt$. 06
- b. If the mean of the following distribution is 16 find m, n and variance 06
- | | | | | | |
|--------|---------------|----|----|---------------|----------------|
| X | 8 | 12 | 16 | 20 | 24 |
| P(X=x) | $\frac{1}{8}$ | m | n | $\frac{1}{4}$ | $\frac{1}{12}$ |
- c. Obtain the Fourier expansion of $f(x) = \left(\frac{\pi-x}{2} \right)^2$ in $(0, 2\pi)$ 08
 Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

- Q.3 a. Find the analytic function $f(z) = u + iv$ in terms of z 06
 if $u + v = e^x (\cos y + \sin y) + \frac{x-y}{x^2+y^2}$
- b. Find the coefficient of regression and hence the equations of the lines of regression for the following data 06

X	78	36	98	25	75	82	90	62	65	39
Y	84	51	91	60	68	62	86	58	53	47

- c. Using convolution theorem Find the inverse Laplace transform of 08
 $\frac{1}{(s^2 + 3s + 13)^2}$

Q.P code
81973

Prog. code
1T00733

- Q.4 a. Obtain Fourier series of $f(x) = |\sin x|$ in $((-\pi, \pi))$. 06
 b. If X denotes the outcome when a fair die is tossed, find the moment generating function of x and hence find the mean and variance of X . 06
 c. Evaluate by using Laplace transforms of $\int_0^\infty e^{-t} (t \int_0^t e^{-4u} \cos u \, du) dt$. 08

- Q.5 a. Find the orthogonal trajectories of family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$. 06
 b. Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. 06
 c. Fit a second-degree parabolic curve to the following data and estimate the Production in 1982. 08

Year (X)	1974	1975	1976	1977	1978	1979	1980	1981
Production (y) (in tons)	12	14	26	42	40	50	52	53

- Q.6 a. Obtain half range Sine series for $f(x) = x - x^2$ in $0 \leq x \leq 1$. 06
 Hence show that $\frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots$
 b. Show that the function $v = e^{2x}(y \cos 2y + x \sin 2y)$ is harmonic. 06
 And find its corresponding analytic function $f(z) = u + iv$.
 c. Find the value of k if the function $f(x) = kx^2(1 - x^3)$, $0 \leq x \leq 1$ 08
 $f(x) = 0$ otherwise.
 Is a probability density function. Also find p ($0 \leq x \leq \frac{1}{2}$) find mean and variance.

